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Uncertain numerals

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1 Abstract

- Goals:
  - Explain a peculiar approximative interpretation associated with numerals that have been marked as uncertain (e.g. *maybe twenty*)
  - Assess predictions of this analysis
  - See what this analysis can tell us about other means of approximation (e.g. *approximately twenty*)
- Results:
  - These approximative peculiarities can be explained through possible world semantics using information associated with numerals
  - This analysis extends to other scalars, yielding correct interpretations
  - This analysis allows us to formalize certain similarities and differences between these uncertain numerals and other means of approximation

2 The phenomenon

- You can use words like *maybe* to mark your uncertainty with respect to an item, and as a result your interlocutor might entertain alternatives to this uncertain item.
  - For example, in (1a) B marks his uncertainty with *maybe*, encouraging A to entertain alternative to *John*, as sketched in (1b).

(1) a. A: Who won the race?
B: Maybe John.
b. {John, Ann, Pete}

- When the uncertain item is a numeral, there is a strong tendency for the set of alternatives to resemble approximation.
  - For example, in (2a) B is likely to think that if the number wasn't 20, then it was some number close to 20, as sketched in (2b).

(2) a. A: How many people competed?
B: Maybe twenty.
b. {18, 19, 20, 21, 22}

- However, this approximative effect does not occur for all uncertain numerals.

  - For example, in (3a) B seems unlikely to think that the bus number must be close to 20 and instead will probably come up with a set of alternatives based on his knowledge of different bus routes.

(3) a. A: Which bus will get me downtown the quickest?
B: Maybe (the) twenty.
b. {20, 6, 77, 15}

- Furthermore, when this approximation effect occurs, the range of alternatives depends on the numeral.
  - For example, replacing *twenty* in (2a) with *twenty-seven* results in a less approximate interpretation

(4) a. A: How many people competed?
B: Maybe twenty-seven.
b. {26, 27, 28}

- This leaves us with the following set of puzzles:

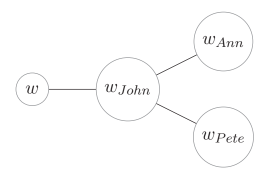
I. Why do uncertain numerals give rise to approximative readings, as in (2)?

II. Why do some uncertain numerals fail to give rise to approximative readings, as in (3)?

III. Why do some uncertain numerals give rise to more approximate readings than others?

3 Analysis

- Consider these phenomena in the context of possible world semantics
- Assume alternatives are possible worlds
  - For example, (1) might look something like:

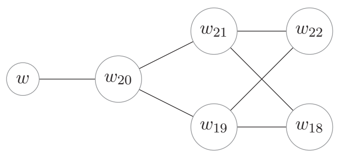


where only the worlds close enough to be plausible are shown.

- Consider the following possible world semantics definitions from Kratzer (1981):
  - Modal base f determines which worlds are accessible from a given world w - accessible worlds are the ones in which all the propositions in f are true

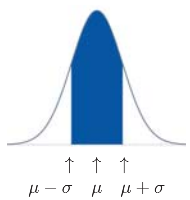
– Ordering source g determines how close the possible worlds are - w_a is as least as close to w as w_b iff all the proposition in f that are true in w_b are also true in w_a

- To explain approximation, we can then assume that numerals contribute information to f and g such that the possible words are those in which nearby numbers are true, as shown below for (3).



- So how do we get the right information into f and g?

- Assume that numerals contribute information about what is close to them in a given context, as in Krifka (2009)
  – Numeral:
    * Represents a range of possible values
    * Probabilities of values are best represented with a normal distribution, as shown below



    * distribution over a number line
    * centered at the uttered numeral (μ)
    * standard deviation (σ) determined pragmatically, involving preference to assign round interpretations (i.e. large σs) to round numerals (see appendix for derivation)
      · Therefore *twenty* will tend to be associated with a larger σ than *twenty-seven*
    * numeral represents the range within σ, other values are too unlikely

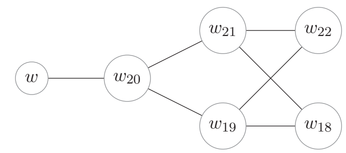
- So, let's assumed numerals are associate with the propositions

$$p_\sigma = \lambda y.y \in \{[\![\mu - \sigma]\!], ..., [\![\mu + \sigma]\!]\}$$
$$p_x = \lambda y.y \in \{[\![\mu - x]\!], ..., [\![\mu + x]\!]\}, 0 \leq x \leq \sigma$$

where, when the numeral is uncertain, $p_\sigma \in f$ and $p_x \in g$. p_σ says that the actual value falls within σ of the value expressed. p_x is a family of propositions for $0 < x < \sigma$ which expresses that values closer to the value expressed are more likely.

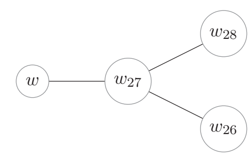
- So now, if we consider (2), we can see how (assuming a σ of 2) the proper interpretation is arrived at. (N.B. $p_{\mathbb{Z}}$ has been included in f since it is presumably understood that we are dealing with whole people.)

$$\mu = 20$$
$$\sigma = 2$$
$$f : p_2 = \lambda y.y \in \{[\![18]\!], ..., [\![22]\!]\}, p_{\mathbb{Z}} = \lambda y.y \in \mathbb{Z}$$
$$g : p_x = \lambda y.y \in \{[\![20 - x]\!], ..., [\![20 + x]\!]\}, 0 < x < 2$$



- We can see that (4) likewise yields a proper interpretation, following from the preference to assign round interpretations to round numbers (as explained in the appendix).

$$\mu = 27$$
$$\sigma = 1 \text{ (2 is blocked)}$$
$$f : p_2 = \lambda y.y \in \{[\![26]\!], ..., [\![28]\!]\}, p_{\mathbb{Z}} = \lambda y.y \in \mathbb{Z}$$
$$g : p_x = \lambda y.y \in \{[\![27 - x]\!], ..., [\![27 + x]\!]\}, 0 < x < 1$$



- Furthermore, on closer inspection we can see that (3), which did not show an approximative effect, does not involve the kind of scalar we've been discussing. Rather, the numeral acts as a label and does not seem to represent a range. Correspondingly we would not expect it to make the same type of contributions to f and g.

- Together, this leads to the following solutions:

I. Uncertain numerals give rise to approximative readings because they introduce p_σ into f and p_x into g, so possible worlds are those in which the numeral is close to the uncertain numeral.

II. Some uncertain numerals fail to give rise to approximative readings because they are not scalar and therefore do not contribute p_σ and p_x

III. Some uncertain numerals give rise to more approximate readings than others because they are associated with larger σs, so p_σ allows more possible worlds.

4 Predictions

- Other words are similar to numerals in that they express ranges which may be best represented by a normal distribution, so they are expected to contribute similar information to f and g when marked as uncertain, resulting in an approximate reading.
- This is indeed the case. For example, when a color term is used scalarly, it gives an approximate reading when combined with *maybe*. The scenario in (5) attempts to induce a scalar reading of *blue*, which leads to alternatives that are hues similar to blue.

(5) a. A: You say you got a good look at John's car. What color is it?
B: Maybe blue.

b. { }

- Colors even show roundness effects, suggesting that they fit into Krifka (2009)'s analysis just like numerals.

(6) a. A: You say you got a good look at John's car. What color is it?
B: Maybe cyan.

b. { }

- In fact, you get approximation with any uncertain scalar. To see this, take any element X, consider its scalar interpretation (e.g. would it would have to mean to make sense in a sentence like *Well, it was only approximately X*, cf. Sauerland and Stateva 2007), and then consider what it would mean under the same interpretation if you marked it as uncertain.
  - Example: Consider a scalar interpretation of Beef Stroganoff, as in *Well, it was only approximately Beef Stroganoff*. Under this same interpretation, in *What Mary cooked was maybe Beef Stroganoff*, you get the reading that what Mary cooked was somewhere near the ideal of Beef Stroganoff, i.e. approximately Beef Stroganoff.

5 Extensions

- If uncertainty markers can act like approximators, then what are true approximators like *approximately*?
  - Instead of involving alternatives, true approximators express that something falls within a range, perhaps with a denotation like

$$[\![\textbf{approximately}]\!] = \lambda n.\lambda y.\exists z \in \{x|\mu_{\mathbf{n}} - \sigma_{\mathbf{n}} \leq x \leq \mu_{\mathbf{n}} + \sigma_{\mathbf{n}}\}|\#y = z$$

(takes a scalar n and some y and returns true if the location of y is within σ of n on the relevant scale)

  - As a result, approximators are less accommodating when it comes to outside information.

(7) It's Susan's birthday today, and she's maybe/#approximately 30.

  - ∗ Here the fact that it is Susan's birthday makes intermediate ages like 31 and 3 months impossible. This restriction cannot be accommodated by range-representing *approximately*, while with *maybe* this restriction can be entered into the modal base f.
  - However, like *maybe* and bare scalars, true approximators show roundness effects (cf. *approximate twenty* vs. *approximately twenty-seven*), which is expected since each determines possible range through σ.

6 Conclusions

- Here we have seen that the peculiar approximative interpretation associated with uncertain numerals can be explained through a possible world semantics with assignment of propositions regarding the numeral's range to f and g.
- Furthermore, this analysis can be successfully applied to other scalar terms.
- Overall this analysis unites different kinds of approximators, including roundness, uncertainty markers, and true approximators, while at the same time providing a source for their differences.

References

Kratzer, A. (1981). The notional category of modality. In H. J. Eikmeyer and H. Rieser (Eds.), *Words, Worlds, and Contexts: New Approaches to Word Semantics*, pp. 38–74. Berlin: de Gruyter.

Krifka, M. (2009). *Approximate Interpretations of Number Words: A Case for Strategic Communication*, pp. 109–132. CSLI Publications.

Sauerland, U. and P. Stateva (2007). Scalar vs. epistemic vagueness: Evidence from approximators. In *Proceedings of SALT 17*, Ithaca, NY. CLC Publications, Cornell University.

A Deriving the round number effect

From Krifka (2009):

- The idea - Strategic communication, in conjuction with a preference for simple expressions (a la Maxim of Manner) and a constraint against expressing some value v with a numeral that does not contain v in its range (a la Maxim of Quality), leads to precise interpretation of complex expressions and approximate interpretations of simple expressions.

- Derivation Part I – OT and the speaker
  - Constraints
    - ∗ SIMPEXP: simple expression > complex expression
    - ∗ INRANGE: The true value of a measure must be in the range of interpretation of the measure term.
  - Result
    - ∗ A value like 40 which corresponds to a simple expression can be expressed using that expression in a precise or round sense (i.e. σ can be small or large).
    - ∗ A value like 39 which corresponds to a relatively complex expression can be expressed either using that expression in a precise sense (small σ) or using a simpler expression in a round sense (large σ), but not by the complex expression in a round sense.

Form/Interpretation Pairs	Value	INRANGE	SIMPEXP
a. ☞ ⟨*forty*, [38...42]⟩	39		
b. ⟨*forty*, [40]⟩	39	*	
c. ⟨*thirty-nine*, [37...41]⟩	39		*
d. ☞ ⟨*thirty-nine*, [39]⟩	39		
e. ☞ ⟨*forty*, [38...42]⟩	40		
f. ☞ ⟨*forty*, [40]⟩	40		
g. ⟨*thirty-nine*, [37...41]⟩	40		*
h. ⟨*thirty-nine*, [39]⟩	40	*	

- Derivation Part II – Game theory and the hearer
  - In Part I we saw that numerals can be ambiguous (*forty* = [38...42] or [40]). How is a hearer to know what level of precision/value the speaker intends? A sketch using strategic communication:
    - ∗ Assume some level of precision α
    - ∗ Under α, the uttered numeral n is considered indistinguishable from other numerals in its range (e.g. if you assume an α such that *forty* is interpreted [38...42], *forty* is indistinguishable from *thirty-eight*, *thirty-nine*, etc. under that same α)

    - ∗ If there is a number in n's range that is simpler, it would have blocked the use of n (as in tableau) → this must not be the correct α, so revise α and try again
    - ∗ When more than one α is possible, choose the largest to increase you chances of including the correct value in your interpretation.
  - Result
    - ∗ More complex expressions will lead the hearer to arrive at smaller αs (and σs) so that their ranges do not include simpler expressions that can block (as in Part I).
    - ∗ Speakers will tend to prefer larger non-blocked αs such that simpler expressions will tend to be interpreted more roundly.

- Summarized computation for complex → precise, simple → approximate (Krifka 2009:117)